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ABSTRACT

Most responses to educational and psychological test items may be represented in binary form. However, such dichotomously scored items present special problems when an analysis of correlational interrelationships among the items is attempted. Two general methods of analyzing binary data are proposed by Horst to partial out the effects of differences in item difficulties: (1) a least square simplex data matrix solution, and (2) a least square simplex covariance matrix solution. Of these, the first was selected for study using (1) a regression approach, (2) a raw data approach, and (3) the computational algorithm for the raw data matrix approach. The results indicate that Horst's modification clearly induces an effect that contaminates the common factor structure of the variables. Further, the findings also indicate that image, alpha, and principal components analysis of correlation matrices obtained from binary data matrices are all satisfactory methods of analysis without the modification. This may be an important finding since it tends to confirm earlier empirical findings concerning the varying difficulties of binary items. (CK)

STUDIES OF HORST'S PROCEDURE FOR BINARY DATA ANALYSIS

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BACKGROUND

Most responses to educational and psychological test items may be represented in binary form. However, such dichotomously scored items present special problems when an analysis of correlational interrelationships among the items is attempted. For example, when a test is intended to measure a unitary trait and also contains items of varying "difficulty," item intercorrelations will not, in general, be homogeneous. The problems of choosing a "proper" coefficient of interrelationship in view of the "contaminating" effect of item "difficulty" appear as yet not to have been solved (Horst, 1965).

Carroll (1961) suggests tetrachoric correlations instead of product-moment or other coefficients because tetrachorics avoid certain problems in varying "difficulty" levels. However, tetrachoric correlations assume "latent" bivariate normal distributions between pairs of items, and it is possible that tetrachoric correlation matrices may not even be Gramian. Horst (1965) and Guttman (1950) strongly criticize the analysis of tetrachoric r's for binary data.

Items with like "difficulty" indices can, in general, be correlated more highly than items with unlike "difficulty" indices. In turn, differences in "difficulties" across items may be represented as extra factors in a factor analysis of the items (Ferguson, 1941; Guttman, 1950; Horst, 1965) which were used as input for the second program.

The second program consisted of a series of three different multivariate methods, each of which can be loosely termed a type of factor analysis. For each set of data, the analyses proceeded both from the residual correlation matrices resulting from Horst's procedure and also from the matrix of phi coefficients computed from the original (permuted) binary data matrices. The number of common factors were held constant for each set of data by inputing to the program the number of Guttman type scales built into that given set of data. Factor analytic methods used were: (1) image analysis following the algorithm developed by Harris (1962); (2) alpha factor analysis following the algorithm given by Kaiser and Caffrey (1965); and (3) principal components following the algorithm presented by Hotelling (1933) and Harmon (1967). Transformations applied to each of the factor solutions were: (1) normal varimax as discussed by Kaiser (1958) and (2) a case II independent cluster solution (Harris and Kaiser, 1964) applied to the principal axis representation of the major product of the initial factor loading matrix for each solution (Pruzek, 1967). To summarize, then, each of four sets of artificial binary data were analyzed with and without Horst's procedure, using each of three factoring procedures with two analytic transformations for each. Thus, twenty-four possible factor pattern matrices were generated for study.

RESULTS AND DISCUSSION

IMAGE ANALYSIS

Binary Data Matrix. In the ensuing discussion, reference will be made to scale factors (factors built into each data set). Data sets A and B have two scale factors: (1) For data set A, scale factor a consists of items 1, 3, 5, 7, 9 and scale factor b consists of items 2, 4, 6, 8, 10. (2) For data set B, scale factor a consists of items 1, 3, 4, 7, 8, 10 and scale factor b consists of items 2, 5, 6, 9. Data sets C and D have three scale factors: (1) For data set C, scale factor a consists of items 1, 3, 4, 8; scale factor b consists of items 7, 9, 11, 12; scale factor c consists of items 2, 5, 6, 10. For data set D, scale factor a consists of items 8, 10, 12; scale factor b consists of items 2, 3, 6, 7; scale factor c consists of 1, 4, 5, 9, 11.

Tables 4, 5, 6, 7 are composed of the analyses for data sets A, B, C, D respectively. The lower case letter next to the difficulty index refers to the particular scale factor into which the item was built. For the artificial data in which two factors were built, two factors were extracted; and for data with three factors, three factors were extracted. An attempt was made to label the columns of the solution matrices with the letters of the scale factors with which they were most closely associated. Thus, if the first column of a varimax solution appears to represent scale factor a for the given data set analyzed, the column is referred to as varimax solution a, and similarly for cluster solution a.

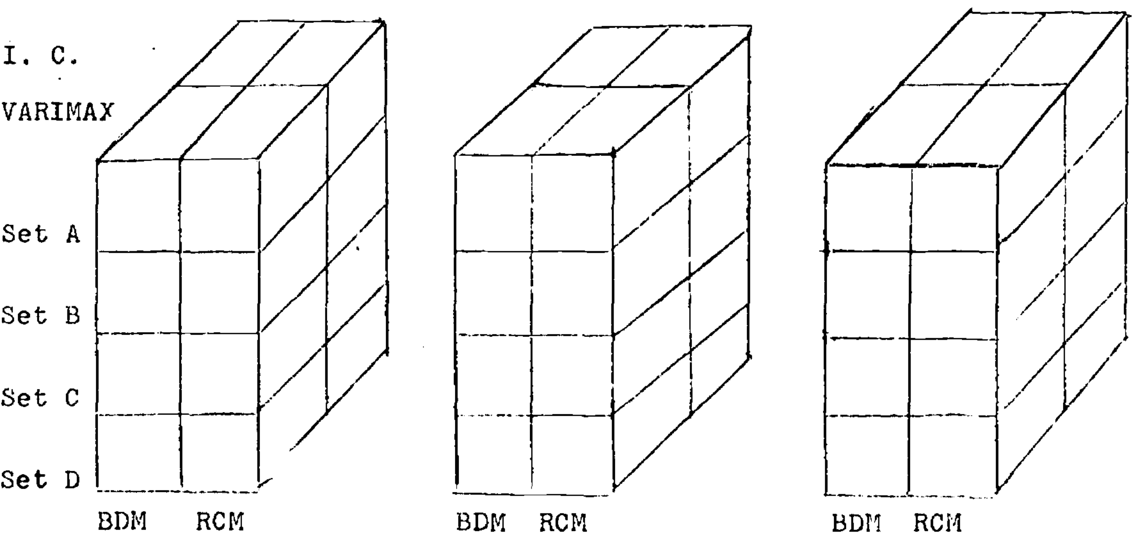

Image Analysis

Alpha Analysis

Principal Components Analysis

The hypothesized scale factors of data sets A, B, and C are clearly well defined by the normal varimax solution. Hypothesized factors defined by the pattern matrix of the independent cluster solution for data sets A and B are clear, with the exception of the negative loading for variable 1 of data set B, which has the highest "difficulty" index of the variables in the set. For data sets C and D, the independent cluster solutions are acceptable; but for data set D, the low negative entries on cluster a are the result of the variables with high difficulty indices on scale factor b and variables with low difficulty indices on scale factor c.

Residual Correlation Matrix. Horst's procedure produced singular matrices for data sets B and C. Because of the singular matrix, image analysis was not applicable. For data sets d D, Horst's procedure resulted in bipolar factors. In an attempt to "partial out" the varying difficulties, Horst's process also produced factorially complex varimax and independent cluster solutions.

For data set A, the varimax and independent cluster solution for the first Harris factor a can be discussed in terms of the difficulty indices of the items of scale factor a and scale factor b. The high positive loadings for varimax solution a and cluster solution a correspond to the items of scale factor a having low difficulty indices, while the high negative loadings correspond to the items of scale factor b having high difficulty indices. Similarly, the high positive loadings of varimax solution b and cluster solution b are associated with those items of scale factor b having low difficulty indices, and the high negative loadings are associated with those items of scale factor a having high difficulty indices.

Both varimax and independent cluster solutions of the Harris factors of data set D have bipolarity in each column of the solution matrices. The high positive loadings of varimax solution a and cluster a are associated with scale factor a, and the high negative loadings are associated with those items of scale factor c having low difficulty indices. For varimax solution b and cluster solution b, the high positive loadings are associated with those items of scale factor b having high indices of difficulty. The high negative loadings on varimax solution b and cluster b are associated with those items of scale factor c having high difficulty indices. The high positive loadings of varimax solution c and cluster solution c are associated with those items of scale factor c having low indices of difficulty while the high negative loadings are associated with those items of scale factor b having difficulty indices around .50.

Horst's procedure produced two singular matrices and rendered the other two data matrices uninterpretable. There was no recovery of factors when using the Horst modification, and the procedure appeared to cause bipolarity and splitting of factors. Data modified by Horst's procedure could not be clearly analyzed through the use of image analysis; however, the unadjusted data analyzed by image analysis was clearly interpretable, and in all four sets of data, recovery of the artificial factors was possible despite widely varying difficulties of items.

ALPHA ANALYSIS

Binary Data Matrix. Alpha factor analysis of the unmodified data for data sets A, B, and C with a normal varimax solution allowed complete recovery of the scale factors. For each set of data, the common factor structure was well defined.

In data set D, the influence of high "difficulty" indices is seen by the loadings of variables 1 and 4 for varimax solution factor a. Although varimax solution factor a is clearly the hypothesized scale factor a, the effects of scale factor c are present.

Alpha analysis with an independent cluster solution, although acceptable and clearly defining the hypothesized factors, tended not to have a clear positive manifold for all clusters; i.e., cluster a for data set A, cluster b for data set B, clusters a, b, and c for data set C, and clusters a and c for data set D.

Residual Correlation Matrix. Horst's modification, as noted in the image section, produced two singular matrices; consequently, no alpha analysis was performed on data sets B and C.

Just as bipolarity occurred in the image analyses, so also did it occur in the alpha analyses. Both the varimax and independent cluster solutions had bipolarity on every factor and cluster.

There was no clear recovery of factors using the Horst modification. The procedure caused bipolarity as well as splitting of the factors. Without the Horst process, the data analyzed by alpha analysis was interpretable; and all four sets of artificial factors were completely recoverable with both a varimax or independent cluster solution.

PRINCIPAL COMPONENTS

Binary Data Matrix. For data sets A, B, and C the hypothesized factors are clearly defined by both varimax and independent cluster solutions. Both transformations for data set D tend to have minor row complexity and lack a clear positive manifold on some factors.

Residual Correlation Matrix. As in the previous analyses, Horst's modification produces bipolarity for each factor and cluster. No factors were clearly recoverable, and each solution was factorially complex. Principal components analysis of the unmodified binary data matrix yielded factors that were clearly the hypothesized factors.

SUMMARY

Horst's modification clearly induces an effect that contaminates the common factor structure of the variables. Analysis of the item difficulties and the factoral structure of the solutions after Horst's modification seem to indicate that an influence due to item difficulties is involved in the common factor distortion. That is, Horst's procedure for partialing out the effects of item difficulty appears to be complicating the common factor structure for the data. The item difficulties appear to have increased effects on the factoral structure rather than decreased effects after the Horst modification.

On the positive side, the findings indicate that image, alpha, and principal components analysis of correlation matrices obtained from binary data matrices are all satisfactory methods of analysis without Horst's modification. This could possibly be an important result; it tends to confirm the empirical findings of Pruzek (1967) and Dingman (1958) that varying difficulties of binary items do not tend to be of great practical consequence, at least when derived clusters are relatively clear.

Research by the authors related to the problem of analysis of binary response data indicates that image analysis generally provides the most interpretable analyses of such data, while alpha analysis tends to produce a Heywood case if several first order partial correlations of a particular variable are relatively high. Principal components analysis has a tendency to produce complex patterns for some data. In view of this, Horst's procedure appears not to warrent further study.

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A P P E N D I X

TABLE 1

Permuted Binary Data Matrix

Individuals	Items 1	2	3	4	5	6	7	8	9	10
1	1									
2	1									
3	1	1								
4		1			1					
5	1	1	1							
6		1			1	1				
7		1			1	1				
8	1		1	1			1			
9	1		1	1			1			
10	1	1	1		1					
11		1			1	1			1	
12	1	1	1	1			1			
13	1	1	1	1	1					
14	1		1	1			1	1		1
15	1		1	1			1	1		1
16	1	1	1	1	1	1				
17	1	1	1	1	1	1	1			
18	1	1	1	1	1	1			1	
19	1	1	1	1	1	1			1	
20	1	1	1	1	1	1			1	
21	1	1	1	1	1		1	1		1
22	1	1	1	1	1	1	1	1		1
23	1	1	1	1	1	1	1	1	1	
24	1	1	1	1	1	1	1	1	1	
25	1	1	1	1	1	1	1	1	1	
26	1	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1

TABLE 2

Simplex Matrix for Table 1

Individuals

	Items 1	2	3	4	5	6	7	8
1								
2								
3								
4								
5	1							
6	1							
7	1	1						
8	1	1	1					
9	1	1	1					
10	1	1	1	1				
11	1	1	1	1				
12	1	1	1	1				
13	1	1	1	1				
14	1	1	1	1	1			
15	1	1	1	1	1	1		
16	1	1	1	1	1	1		
17	1	1	1	1	1	1		
18	1	1	1	1	1	1		
19	1	1	1	1	1	1	1	
20	1	1	1	1	1	1	1	
21	1	1	1	1	1	1	1	
22	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1

TABLE 3

Binary Data Matrix for Table 1

Individuals	Items 1	2	3	4	5	6	7	8	9	10
1							1	1	1	
2							1	1	1	1
3							1	1		
4							1	1	1	
5	1									
6	1									
7	1						1			
8	1	1					1			
9	1	1					1	1		
10	1	1	1				1	1	1	1
11	1	1	1				1	1	1	1
12	1	1	1				1	1		
13	1	1	1				1	1	1	
14	1	1	1				1	1	1	1
15	1	1	1	1						
16	1	1	1	1			1			
17	1	1	1	1						
18	1	1	1	1			1	1	1	
19	1	1	1	1	1		1	1	1	1
20	1	1	1	1	1		1	1	1	1
21	1	1	1	1	1		1	1	1	1
22	1	1	1	1	1	1	1	1		
23	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1				
25	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1	1	
28	1	1	1	1	1	1				
29	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1

TABLE 4

DATA SET A

Variable	Difficulty	Scale Factor	IMAGE B. D. M. VARIMAX a	IMAGE B. D. M. VARIMAX b	IMAGE R. C. M. VARIMAX b	IMAGE R. C. M. VARIMAX a	ALPHA B. D. M. VARIMAX b	ALPHA B. D. M. VARIMAX a	ALPHA R. C. M. VARIMAX a	ALPHA R. C. M. VARIMAX b	PRINCIPAL COMPONENTS B. D. M. VARIMAX a	PRINCIPAL COMPONENTS B. D. M. VARIMAX b	PRINCIPAL COMPONENTS R. C. M. VARIMAX a	PRINCIPAL COMPONENTS R. C. M. VARIMAX b
1	.87	a	.65	.13	-.47	.25	.15	.62	.46	.18	.73	.15	.58	.18
2	.80	b	.18	.63	.02	-.63	.64	.18	-.03	-.61	.18	.72	.03	-.76
3	.77	a	.86	-.11	-.69	.50	-.12	.91	.82	.44	.90	-.16	.77	.43
4	.70	b	.08	.76	.07	-.72	.81	.05	-.07	-.82	.05	.84	-.06	-.33
5	.70	a	.89	.07	-.68	.52	.07	.90	.66	.53	.91	.07	.71	.51
6	.57	b	.11	.81	.58	-.57	.87	.10	-.64	-.55	.10	.88	-.66	-.54
7	.53	a	.76	.06	-.30	.65	.04	.84	.27	.71	.83	.03	.32	.73
8	.40	b	.03	.82	.80	-.04	.82	.02	-.76	-.06	.02	.85	-.85	-.01
9	.40	a	.63	.21	-.17	.70	.21	.63	.21	.62	.71	.21	.22	.72
10	.30	b	-.02	.76	.85	.08	.69	.03	-.83	.08	-.00	.78	-.88	.11

Variable	Difficulty	Scale Factor	IMAGE INDEPENDENT CLUSTER b	IMAGE INDEPENDENT CLUSTER a	IMAGE INDEPENDENT CLUSTER a	IMAGE INDEPENDENT CLUSTER b	ALPHA INDEPENDENT CLUSTER b	ALPHA INDEPENDENT CLUSTER a	ALPHA INDEPENDENT CLUSTER b	ALPHA INDEPENDENT CLUSTER a	PRINCIPAL COMPONENTS INDEPENDENT CLUSTER b	PRINCIPAL COMPONENTS INDEPENDENT CLUSTER a	PRINCIPAL COMPONENTS INDEPENDENT CLUSTER a	PRINCIPAL COMPONENTS INDEPENDENT CLUSTER b
1	.87	a	.09	.65	.22	-.39	.22	.57	-.46	.07	.11	.72	.57	.07
2	.80	b	.63	.12	-.73	-.27	.66	.01	-.18	-.69	.72	.12	.26	-.84
3	.77	a	-.17	.88	.46	-.51	-.02	.92	-.77	.26	-.17	.92	.70	.31
4	.70	b	.77	.00	-.82	-.26	.82	-.17	-.20	-.91	.85	-.03	.19	-.90
5	.70	a	.01	.89	.50	-.49	.17	.86	-.56	.41	.01	.91	.62	.40
6	.57	b	.82	.02	-.57	.37	.88	-.13	.52	-.44	.88	.02	[illegible]	-.45
7	.53	a	.01	.76	.70	-.03	.13	.78	-.07	.72	-.02	.84	[illegible]	.73
8	.40	b	.82	-.05	.08	.84	.83	-.19	.83	.15	.86	-.06	-.91	.18
9	.40	a	.17	.61	.79	.14	.28	.56	-.03	.64	.16	.70	.02	.75
10	.30	b	.77	-.11	.22	.95	.70	-.15	.96	.34	.79	-.08	-.98	.31

TABLE 5

DATA SET B

Variable	Difficulty	Scale Factor	IMAGE B. D. M. VARIMAX a	IMAGE B. D. M. VARIMAX b	IMAGE R. C. M. VARIMAX	ALPHA B. D. M. VARIMAX a	ALPHA B. D. M. VARIMAX b	ALPHA R. C. M. VARIMAX	PRINCIPAL COMPONENTS B. D. M. VARIMAX a	PRINCIPAL COMPONENTS B. D. M. VARIMAX b	PRINCIPAL COMPONENTS R. C. M. VARIMAX b	PRINCIPAL COMPONENTS R. C. M. VARIMAX a
1	.87	a	.66	-.26		.61	-.22		.72	-.28	.06	.90
2	.80	b	-.10	.73		-.10	.72		-.12	.82	-.57	-.10
3	.77	a	.79	.10		.78	.10		.82	.11	.02	.92
4	.70	a	.83	.18		.84	.17		.86	.18	.04	.63
5	.70	b	.02	.86	SINGULAR MATRIX	.00	.94	SINGULAR MATRIX	.01	.92	-.48	-.58
6	.57	b	.14	.80		.13	.88		.14	.89	-.70	-.60
7	.53	a	.83	-.00		.84	-.01		.87	-.02	.74	.20
8	.40	a	.75	.26		.79	.26		.79	.26	.94	-.05
9	.40	b	.31	.63		.32	.64		.33	.72	-.54	-.37
10	.30	a	.64	.11		.61	.09		.70	.03	.78	-.10

Variable	Difficulty	Scale Factor	INDEPENDENT CLUSTER a	INDEPENDENT CLUSTER b	INDEPENDENT CLUSTER	INDEPENDENT CLUSTER a	INDEPENDENT CLUSTER b	INDEPENDENT CLUSTER	INDEPENDENT CLUSTER a	INDEPENDENT CLUSTER b	INDEPENDENT CLUSTER b	INDEPENDENT CLUSTER a
1	.87	a	.68	-.34		.65	-.29		.74	-.35	-.11	.93
2	.80	b	-.15	.75		-.14	.74		-.17	.84	-.58	-.00
3	.77	a	.79	.01		.78	.01		.82	.03	-.15	.96
4	.70	a	.83	.09		.85	.07		.86	.10	-.08	.65
5	.70	b	-.04	.87	SINGULAR MATRIX	-.06	.95	SINGULAR MATRIX	-.05	.93	-.39	-.52
6	.57	b	.08	.80		.03	.87		.08	.90	-.62	-.50
7	.53	a	.85	-.10		.86	-.11		.87	-.10	.74	.08
8	.40	a	.74	.18		.79	.17		.78	.19	.99	-.22
9	.40	b	.27	.60		.28	.61		.29	.70	-.50	-.29
10	.30	a	.65	.04		.61	.02		.70	.02	.83	-.25

TABLE 6

DATA SET C

Variable	Scale Factor	Diff.	IMAGE B. D. M. VARIMAX a	b	c	IMAGE R. C. M. VARIMAX	ALPHA B. D. M. VARIMAX a	c	b	ALPHA R. C. M. VARIMAX	PRINCIPAL COMPONENTS B. D. M. VARIMAX a	c	b	PRINCIPAL COMPONENTS R. C. M. VARIMAX a	b	c
1	a	.87	.69	.26	.03		.67	.02	.26		.75	.03	.28	.70	.23	.20
2	c	.80	.03	.17	.72		.00	.68	.15		.01	.79	.16	-.28	-.86	-.04
3	a	.77	.90	.05	-.15		.95	-.15	.06		.92	-.15	.06	.94	.00	.01
4	a	.70	.88	.08	-.08		.89	-.06	.04		.91	-.07	.05	.88	-.21	-.03
5	c	.70	.12	.15	.84		.16	.92	.14		.14	.90	.15	-.12	-.78	.05
6	c	.57	-.04	.06	.83	SINGULAR	-.01	.90	.08	SINGULAR	-.02	.90	.07	.03	.02	.79
7	b	.53	.37	.53	.01	MATRIX	.39	.01	.50	MATRIX	.41	-.01	.62	-.23	.80	-.15
8	a	.53	.70	.02	.13		.70	.11	.04		.80	.14	.00	.66	.14	-.34
9	b	.40	.08	.85	-.05		.10	-.08	.97		.08	-.07	.93	-.39	.68	.01
10	c	.40	-.13	-.05	.68		-.15	.67	-.05		-.15	.73	-.08	.01	-.15	.63
11	b	.30	.02	.89	.15		.01	.17	.96		.00	.15	.93	-.63	.28	.23
12	b	.20	.08	.73	.24		.09	.24	.65		.06	.24	.79	-.19	.05	.74

Variable	Scale Factor	Diff.	INDEPENDENT CLUSTER a	c	b	INDEPENDENT CLUSTER	INDEPENDENT CLUSTER a	c	b	INDEPENDENT CLUSTER	INDEPENDENT CLUSTER a	c	b	INDEPENDENT CLUSTER a	b	c
1	a	.87	.64	.05	.16		.63	.06	.16		.71	.04	.18	.73	.29	.25
2	c	.80	.00	.77	.09		-.03	.72	.06		-.00	.83	.08	-.28	-.88	-.09
3	a	.77	.38	-.10	-.06		.93	-.08	-.05		.91	-.12	-.05	.95	.06	.06
4	a	.70	.85	-.03	-.04		.88	.00	-.08		.90	-.03	-.07	.89	-.14	.00
5	c	.70	.10	.91	.04		.11	1.00	.00		.13	.96	.04	[illegible]	[illegible]	.01
6	c	.57	-.05	.91	-.04	SINGULAR	-.05	.97	-.03	SINGULAR	-.03	.95	[illegible]	[illegible]	[illegible]	.80
7	b	.53	.30	-.03	.48	MATRIX	.33	-.01	.45	MATRIX	.34	-.05	[illegible]	[illegible]	[illegible]	-.14
8	a	.53	.59	.20	-.10		.68	.17	-.07		.80	.20	[illegible]	[illegible]	[illegible]	-.31
9	b	.40	-.03	-.15	.85		-.01	-.17	.97		-.03	-.17	.93	[illegible]	.05	.01
10	c	.40	-.13	.74	-.12		-.16	.72	-.11		-.14	.84	-.14	.07	-.14	.64
11	b	.30	-.09	.05	.88		-.10	.10	.94		-.10	.08	.92	-.62	.23	.21
12	b	.20	-.01	.17	.69		.00	.20	.62		-.03	.19	.76	-.12	.05	.74

TABLE 7

DATA SET D

Variable	Scale Factor	Diff.	IMAGE B. D. M. VARIMAX c	b	a	IMAGE R. C. M. VARIMAX b	a	c	ALPHA B. D. M. VARIMAX c	b	a	ALPHA R. C. M. VARIMAX b	a	c	PRINCIPAL COMPONENTS B. D. M. VARIMAX c	b	a	PRINCIPAL COMPONENTS R. C. M. VARIMAX b	a	c
1	c	.80	.54	.00	.37	-.46	.00	-.01	.56	.00	.32	-.56	-.01	-.02	.64	.02	.36	.10	-.08	.74
2	b	.77	.00	.81	-.08	.80	-.03	-.20	.93	.77	-.07	.75	-.02	-.25	.01	.87	-.10	.29	-.04	-.80
3	b	.70	-.16	.86	-.10	.77	-.01	-.38	-.17	.92	-.12	.70	-.01	-.44	-.17	.91	-.11	.49	-.04	-.71
4	c	.70	.66	.03	.44	-.51	.17	.17	.76	.04	.41	-.64	.12	.13	.76	.05	.42	-.13	.15	.71
5	c	.57	.78	-.14	.25	-.56	-.10	.55	.83	-.13	.20	-.55	-.07	.59	.85	-.13	.21	-.65	-.08	.53
6	b	.53	-.23	.79	.22	.45	-.01	-.70	-.20	.86	.23	.35	-.03	-.80	-.22	.85	.23	.82	-.06	-.33
7	b	.40	-.02	.69	.24	.18	-.31	-.70	.00	.67	.23	.29	-.20	-.57	.00	.77	.24	.71	-.33	-.20
8	a	.40	.20	.06	.80	-.02	.78	-.11	.23	.05	.81	-.10	.73	-.14	.21	.04	.87	.17	.84	-.09
9	c	.40	.86	-.11	.00	-.08	-.32	.82	.89	-.11	-.03	.01	-.21	.93	.90	-.10	-.03	-.90	-.26	-.03
10	a	.30	.08	.05	.87	-.19	.79	-.13	.06	.04	1.00	-.09	1.11	-.03	.07	.04	.94	-.10	.91	.15
11	c	.30	.74	-.19	.03	-.14	-.45	.69	.69	-.19	.04	-.07	-.44	.72	.80	-.19	-.01	-.77	-.43	.03
12	a	.20	.20	.09	.70	.01	.40	.02	.21	.11	.65	.02	.38	-.00	.19	.09	.79	-.08	.62	-.11

Variable	Scale Factor	Diff.	INDEPENDENT CLUSTER c	b	a	INDEPENDENT CLUSTER b	a	c	INDEPENDENT CLUSTER c	b	a	INDEPENDENT CLUSTER c	a	b	INDEPENDENT CLUSTER c	b	a	INDEPENDENT CLUSTER c	a	b
1	c	.80	.51	.05	.28	-.55	-.05	-.22	.56	.04	.21	-.30	-.11	-.67	.63	.07	.26	.35	.15	-.85
2	b	.77	.15	.82	-.16	.86	.02	.13	.11	.77	-.15	.08	.04	.82	.14	.88	-.18	.08	-.03	.79
3	b	.70	-.03	.85	-.15	.78	-.00	-.08	-.02	.92	-.19	-.16	-.01	.72	-.05	.90	-.17	.33	-.06	.64
4	c	.70	.65	.08	.32	-.51	.18	-.01	.76	.09	.25	-.16	.06	-.70	.74	.10	.30	.05	.14	.72
5	c	.57	.80	-.05	.09	-.49	-.07	.35	.86	-.05	.03	.39	-.03	-.52	.87	-.04	.06	-.54	-.04	-.44
6	b	.53	-.20	.72	.22	.29	-.11	-.60	-.13	.81	.20	-.73	-.17	.23	-.18	.80	.22	.81	-.14	.15
7	b	.40	.02	.65	.20	-.08	-.48	-.77	.06	.65	.18	-.51	-.30	.19	.05	.75	.19	.75	-.42	-.00
8	a	.40	.02	-.03	.84	.09	.88	.03	.10	-.01	.82	-.14	.73	-.05	.04	-.03	.89	.12	.86	-.01
9	c	.40	.95	.04	-.20	.12	-.22	.83	.98	.00	-.24	1.01	-.01	.19	.97	.02	-.21	-.95	-.17	.20
10	a	.30	-.14	-.06	.94	-.12	.88	-.07	-.14	-.06	1.07	.02	1.17	.03	-.12	-.06	.99	.06	.94	-.05
11	c	.30	.80	-.07	-.13	-.02	-.40	.63	.72	-.11	-.11	.73	-.32	.01	.84	-.09	-.15	-.78	-.37	.08
12	a	.20	.06	.02	.72	.09	.47	.11	.12	.07	.64	.04	.41	.08	.04	.02	.81	-.19	.67	.23